Tottenham and Bennett
CONSULTING ENGINEERS

TECHNICAL NOTE: 2

Design of a simply supported beam of uniform cross-section which is subject to a load which reduces linearly with deflection

Consider a beam which is subject to a loading pattern which may be defined by the equation q = qo - kxy where q is the load (in kN/m), qo is a constant value of loading, y is the deflection at any point and k is a constant. This may be solved by setting y as a Fourier Series, calculating q as EI times the fourth derivative of y with respect to length, setting qo as a square wave Fourier Series and equating the relevant coefficients.

Below we calculate the maximum values of Bending Moment and Deflection (which occur at midspan) and consider the first 100 coefficients in each Fourier Series.

If y = A sin (πx/L) + B sin (3πx/L) + C sin (5πx/L) + D sin (7πx/L) + ...

q = EI x (A (π/L)^4 sin (πx/L) + B (3π/L)^4 sin (3πx/L) + C (5π/L)^4 sin (5πx/L) + ...

qo = (4/π) x qo x (sin (πx/L) - (sin (3πx/L))/3 + (sin (5πx/L))/5 - ....

Setting k = ϕ (π/L)^4 x EI gives

A = 4qo x L^4 / (π^5 EI x (1 + ϕ)   B = - 4qo x L^4 / (3π^5 EI x (3^4 +ϕ)

and hence

y = ((4qo x L^4) / (EI x π^5)) x Σ (sin ((2n + 1)πx / 2L) / ((2n+ 1) x ((2n + 1)^4 + 1)

$k := 2\cdot\frac{kN}{m}\cdot\frac{1}{mm}$ $k = 2\,MPa$ $l := 3.15\cdot m$ $E := 205\cdot GPa$ $I := 1750\cdot cm^4$ $qo := 10\cdot\frac{kN}{m}$

Let $\phi := \frac{k\cdot l^4}{\pi^4\cdot E\cdot I}$ $\phi = 0.563$ $Q_{totrigid} := qo\cdot l$

Minimum Load per unit length is given by

$$q_{min} := \frac{4\cdot qo}{\pi}\cdot\left[\sum_{n=0}^{100}\frac{(2n+1)^3}{\left[(2n+1)^4+\phi\right]}\cdot\sin\left[(2n+1)\cdot\frac{\pi}{2}\right]\right] \qquad q_{min} = 5.47\,\frac{kN}{m} \qquad \frac{q_{min}}{qo} = 0.547$$

Total Load on beam is given by

$$Q_{tot} := \frac{8\cdot qo\cdot l}{\pi^2}\cdot\left[\sum_{n=0}^{100}\frac{(2n+1)^2}{\left[(2n+1)^4+\phi\right]}\right] \qquad Q_{tot} = 22.214\,kN \qquad \frac{Q_{tot}}{Q_{totrigid}} = 0.705$$

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Maximum Bending Moment, M is given by

$$M := \frac{4 \cdot q_o \cdot l^2}{\pi^3} \cdot \left[\sum_{n=0}^{100} \frac{(2n+1)}{\left[(2n+1)^4 + \phi\right]} \cdot \sin\left[(2n+1) \cdot \frac{\pi}{2}\right] \right] \qquad M = 7.793\,kN \cdot m$$

Maximum deflection, is given by

$$\Delta := \frac{4 \cdot q_o \cdot l^4}{\pi^5 \cdot E \cdot I} \cdot \sum_{n=0}^{100} \frac{1}{\left[(2n+1)^4 + \phi\right] \cdot (2n+1)} \cdot \sin\left[(2n+1) \cdot \frac{\pi}{2}\right] \qquad \Delta = 2.281\,mm$$

$$\Delta rigid := \frac{5 \cdot q_o \cdot l^4}{384 \cdot E \cdot I} \qquad \Delta rigid = 3.573\,mm \qquad \frac{\Delta}{\Delta rigid} = 0.638$$

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