Local Bending of Beam Flange due to Single Point Load

1 Introduction

In this note we calculate the forces required to cause local bending failure of flanges using traditional yield-line theory and compare the values obtained with those obtained using other methods. Initially we consider cases when there is no direct stress in the flange but then go on to consider cases where the flange is subject to direct stress.

The following non-dimensional variables are used throughout

Ratio of distance of bolt from yield-line by web

to distance of edge of flange from yield-line by web = ϕ

Direct flange stress as a proportion of yield stress = σf

Rounding factors for straight-line yield-line patterns = ϒ

(reduction factor applied to yield-lines between bolt

and web to allow for rounding effect)

Force required to cause plastic failure of the flange expressed

as a multiple of the fully plastic moment per unit length of the flange = Pm

Where formulae derived from yield-line theory contain unknown dimensions Pm has been partially differentiated with respect to these dimensions to obtain minimum values of Pm

2 Alternative Yield Line Mechanisms

The following alternative mechanisms are considered (see figure 1 below)

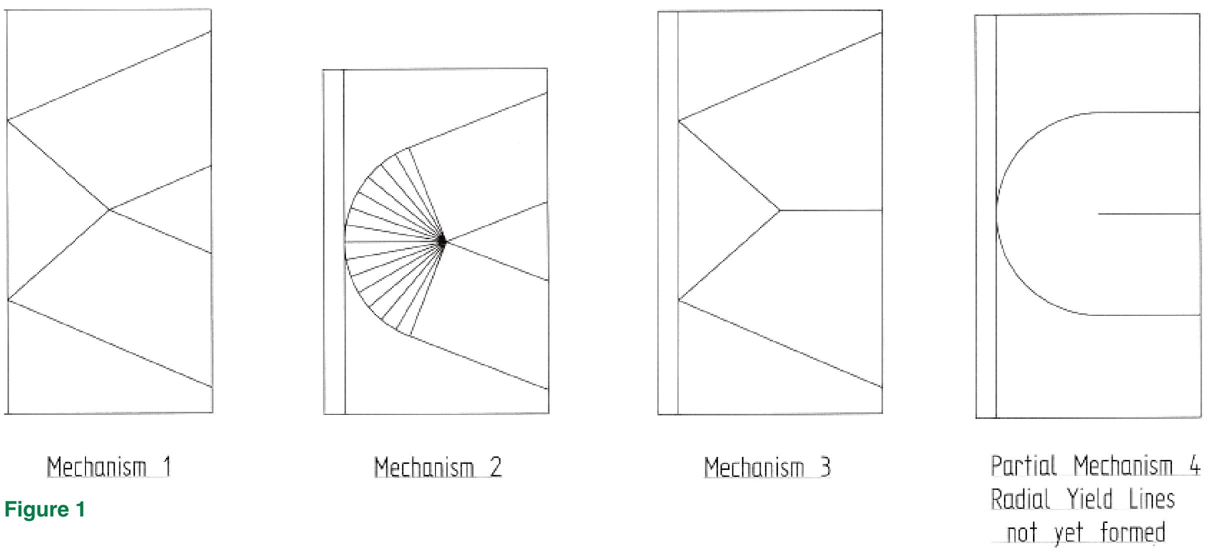

Figure 1

Mechanism 1 - 7 hinges allowing for prying action

$$P_{m1} := \overrightarrow{\left[2\cdot\left(1-\sigma_f^{\,2}\right)^{0.5}\cdot\left[\frac{\gamma}{\phi}\cdot\left[16\cdot(1-\phi)+7\cdot\phi\cdot\gamma\right]\right]^{0.5}\right]}$$ **Eqn 1**

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Mechanism 2 - 4 straight hinges and part of a circular "cone"

$$\theta := \operatorname{acos}\left[\overrightarrow{\frac{\left[\phi - 1 + \left(3\cdot\phi^2 - 2\cdot\phi + 1\right)^{0.5}\right]}{2\cdot\phi}}\right] \quad \text{Eqn 2}$$

$$P_{m2} := \overrightarrow{\left[2\left[\left(1 - \sigma_f^{\,2}\right)^{0.5}\right]\cdot\left(\frac{2 - 2\phi + \phi\cdot\cos(\theta)}{\phi\cdot\sin(\theta)} + 2\cdot\theta\right)\right]} \quad \text{Eqn 3}$$

Note that equations 2 and 3 are approximate. θ above gives the minimum value of Pm2 when σf = 0 and the reduction factor from mechanisms 1 and 3 has been applied to the value of Pm3 for σf = 0.

Precise solutions are given below and may be obtained by setting Xa equal to zero in equation 5, solving for and then substituting this value in equation 4.

$$P_{m2ex} := \overrightarrow{\left[2\cdot\left[\frac{2 - 2\phi + \phi\cdot\cos(\theta)}{\phi\cdot\sin(\theta)}\cdot\left[\left(1 - \sigma_f^{\,2}\right)\cdot(\cos(\theta))^2 + (\sin(\theta))^2\right] + 2\cdot\theta\cdot\left(1 - \frac{\sigma_f^{\,2}}{2}\right)\right]\right]} \quad \text{Eqn 4}$$

$$X_a := \overrightarrow{\left[(1 - \phi + \phi\cdot\cos(\theta))\cdot\cos(\theta)\cdot\left(1 - 2\cdot\sigma_f^{\,2} + \sigma_f^{\,2}\cdot\cos(\theta)^2\right) - \frac{\phi\cdot\left(1 - \sigma_f^{\,2}\right)}{2}\right]} \quad \text{Eqn 5}$$

Mechanism 3 - 6 hinges - No prying action

$$P_{m3} := \overrightarrow{\left[2\left(1 - \sigma_f^{\,2}\right)^{0.5}\left[\frac{2\cdot\gamma\cdot[1 - \phi\cdot(1 - \gamma)]\cdot[8 - \phi\cdot(8 - 7\cdot\gamma)]}{\phi\cdot[1 + \phi\cdot(2\cdot\gamma - 1)]}\right]^{0.5}\right]} \quad \text{Eqn 6}$$

Partial Mechanism 4

$$P_{m4} := \overrightarrow{\left[\pi\cdot\left(1 - \frac{\sigma_f^{\,2}}{2}\right) + \frac{4\cdot(1 - \phi)}{\phi}\cdot\left(1 - \sigma_f^{\,2}\right)\right]} \quad \text{Eqn 7}$$

3 Formulae proposed by others

Owens & Cheal Formula

$$P_{mOWCH} := \overrightarrow{\left[4\cdot\frac{(1 - \phi)}{\phi}\cdot\left(1 - \sigma_f^{\,2}\right) + 5.6\cdot\left(1 - \frac{\sigma_f^{\,2}}{2}\right)\right]} \quad \text{Eqn 8}$$

SCI - " Green Book " Formula (after Zoetermijer for partial prying)

$$P_{mGB} := \overrightarrow{\left(\frac{2.5 + 5.5\cdot\phi}{\phi}\right)} \quad \text{Eqn 9}$$

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EC3 formula

$$RF_{EC31} := \frac{\left(\frac{148}{110} - \sigma_f\right)}{\left(\frac{76}{110}\right)}$$ **Eqn 10**

$$RF_{EC3} := \mathrm{if}(RF_{EC31} > 1, 1, RF_{EC31})$$ **Eqn 11**

$$P_{mEC3} := RF_{EC3} \cdot P_{mGB}$$ **Eqn 12**

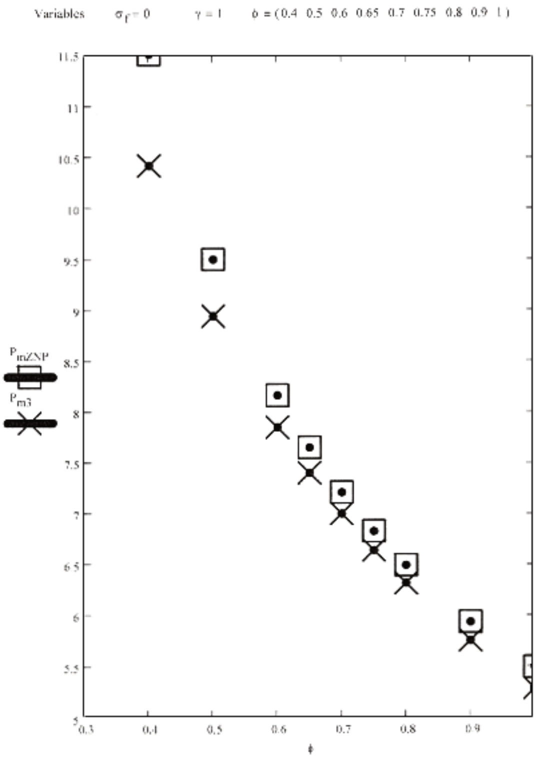
Eqn 5

Steel Designer's Manual
(6th Edition) Formula (after Zoetermijer for no prying)

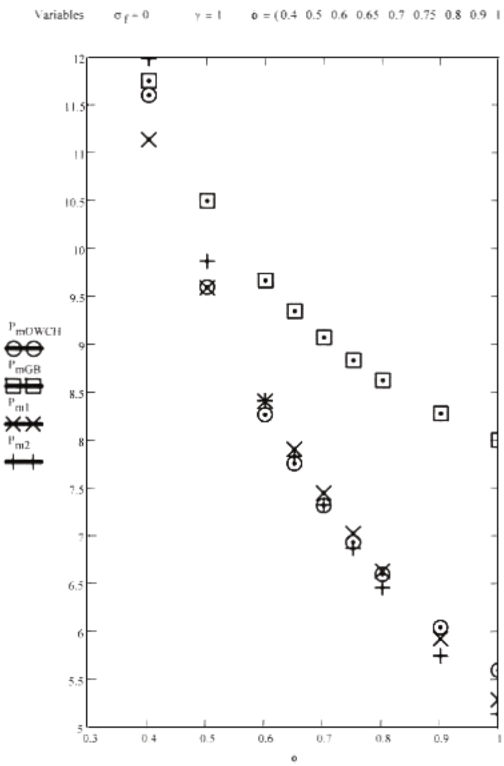
Eqn 5

$$P_{mZNP} := \overrightarrow{\left(\frac{4 + 1.5 \cdot \phi}{\phi}\right)}$$ **Eqn 13**

4 Design Curves for zero direct stress and no prying



5 Design Curves for zero direct stress and prying



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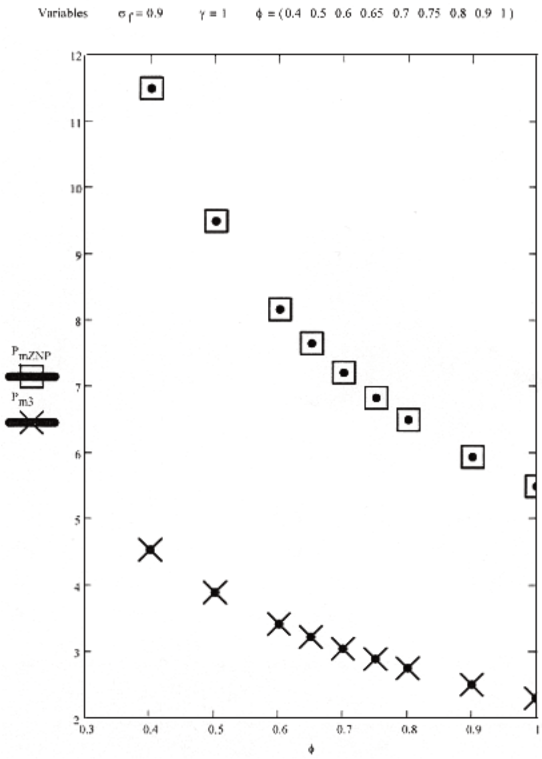
6 Design Curves for direct stress = 90 % of yield stress and no prying



7 Design Curves for direct stress = 90 % of yield stress and prying